Proof Search and Proof nets Construction in Linear Logic

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In the context of the design of correct software, some proposals are based on the following programming paradigms: *proof-as-programs* and *proof-search-as-computation*. The first one corresponds to the Curry-Howard isomorphism that associates a λ-term to a proof, mainly in intuitionistic logic. From a given specification, expressed in the given logic, we try to construct a proof of it and to extract a program from this proof. The second paradigm focuses also on the proof construction, the proof search process corresponding to computation. Thus the common problem is to be able to develop or construct proofs in the given logic. Here we consider linear logic (LL) that has, through proof construction, some applications to computation as well for concurrent logic programming [5, 12] or functional programming [3] as for automated theorem proving (dedicated for instance to prove program properties).

Many works on linear logic and their applications to computer science [1] involve methods and techniques to deal with the problem of proof construction in some linear sequent calculi. In this context we have previously proposed algorithms and techniques for proof construction in various fragments of linear logic [9], mainly based on the definition of canonical proofs that are complete w.r.t. the provability. In parallel, it was natural to consider provability in linear logic through proof net construction [8]. This notion of *proof net* [10] was introduced as a new kind of syntax of linear logic in order to cope with the problems arising from the intrinsic parallelism of linear sequent calculus and to also analyze some algorithmic aspects of proof search. Proof nets allow a compact and essential representation of the logic apparently better than sequent calculus. Naturally defined for the multiplicative linear logic (MLL), this notion has been defined or extended for multiplicative and additive linear logic (MALL) and for other logics [10]. Proof nets construction can be considered with two different approaches. A first one consists in constructing so-called proof structures and in verifying afterwards, with different possible criteria [2], if such structures are proof nets. A second one consists in constructing step by step a proof structure having properties such that the final structure we obtain is surely a proof net. This way to deal with proof nets appears essential for the automation of proof net construction for a given formula or sequent and a first algorithm for the MLL fragment, based on such ideas, has been proposed in [8]. Our purpose, here, after analyses, critics and improvements of this initial work consists in proposing common basic principles for the design of algorithms for automated proof nets construction in linear logic fragments. They come from the analysis of proof nets notion through our results on proof search in linear sequent calculus [9]. After the definition of these principles (inductive construction, connection principle), we propose a new algorithm for MLL proof nets, more efficient and deterministic, and prove its correctness and completeness. Then we can consider the relationship with proof search in linear sequent calculi. From this approach of proof nets construction, it is possible to extend or specialize its principles to deal with more significant logical fragments including for instance the additive connectives.

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Regarding recent proposals for MALL proof nets definition [10] and for proof net construction [11] (that is presented as an extension of [8]), we can define a new algorithm for MALL proof nets construction and prove its essential properties [7].
In further works, we will study the deep relationships with our results about proof search in the MALL fragment with specific management of the additive connectives [6] and also with our recent investigations on connection methods and their applications to proof nets construction [4].

References

[1] V. Alexiev. Applications of Linear Logic to Computation: An Overview. *Bulletin of the IGPL*, 2(1):77–107, 1994.

[2] A. Asperti and G. Dore. Yet another correctness criterion for multiplicative linear logic with mix. In *Logic at St Petersburg '94, Symposium on Logical Foundations of Computer Science, LNCS 813*, pages 34–46, St Petersburg, Russia, July 1994.

[3] N. Benton, G. Bierman, V. de Paiva, and M. Hyland. A term calculus for intuitionistic linear logic. In *Int. Conference on Typed Lambda Calculi and Applications, LNCS 664*, pages 75–90, Utrecht, The Netherlands, March 1993.

[4] D. Galmiche. Connection methods and proof nets construction in linear logic fragments (abstract). In *CADE-13 Workshop on Proof Search in Type-Theoretic Languages*, New Brunswick, NJ, USA, July 1996.

[5] D. Galmiche and E. Boudinet. Proofs, concurrent objects and computations in a FILL framework. In *Workshop on Object-based Parallel and Distributed Computation, OBPDC'95, LNCS 1107*, pages 148–167, Tokyo, Japan, 1996.

[6] D. Galmiche and J.Y. Marion. Dealing with additives in MALL proof search. In *2nd Workshop on Non-standard Logics and Logical Aspects of Computer Science*, Irkutsk, Russia, June 1995.

[7] D. Galmiche and B. Martin. Automated proof nets construction in multiplicative and additive linear logic. Technical report, CRIN-CNRS, 1997.

[8] D. Galmiche and G. Perrier. A procedure for automatic proof nets construction. In *LPAR'92, International Conference on Logic Programming and Automated Reasoning, LNAI 624*, pages 42–53, St. Petersburg, Russia, July 1992.

[9] D. Galmiche and G. Perrier. Foundations of proof search strategies design in linear logic. In *Logic at St Petersburg '94, Symposium on Logical Foundations of Computer Science, LNCS 813*, pages 101–113, St Petersburg, Russia, July 1994.

[10] J.Y. Girard. Proof nets: the parallel syntax for proof theory. In Ursini and Agliano, editors, *Logic and Algebra*, New York, 1995. M. Dekker.

[11] S. Matsuoka. Linear proof search as proof net construction: the multiplicative additive case (extended abstract). In J. Harland & D. Pym, editor, *ILPS'95 Workshop on Operational & Denotational Semantics of Logic Programming: Extending Proof- and Model-theoretic Analyses*, Portland, Oregon, December 1995.

[12] D. Miller. A multiple-conclusion meta-logic. In *9th IEEE Symposium on Logic in Computer Science*, pages 272–281, Paris, France, July 1994.